DMD-based estimation of the flow field behind a thin airfoil at high angles

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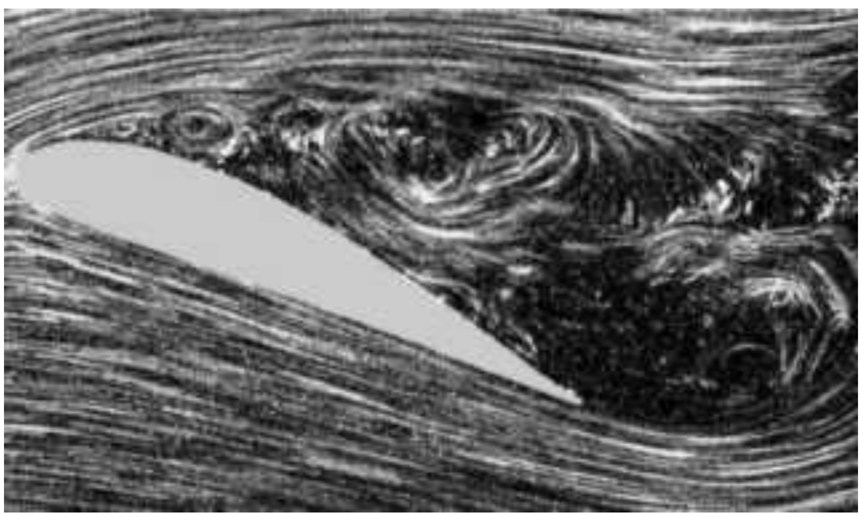
Airfoils at high angles of attack

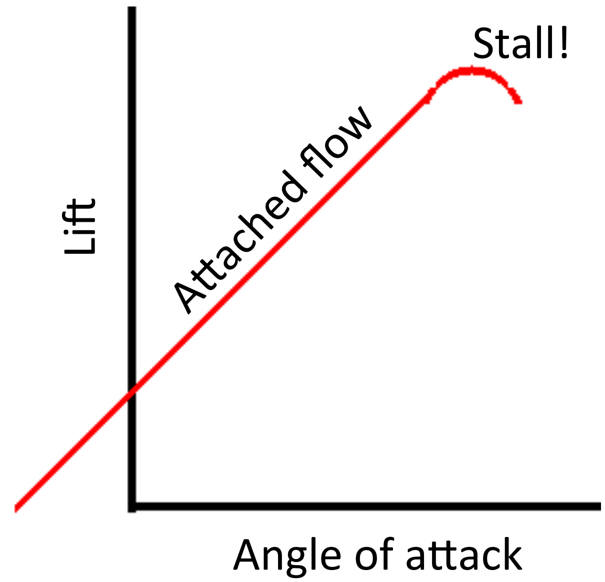
Low angle of attack

Attached flow at a low angle of attack (Source: www.grc.nasa.gov)

High angle of attack



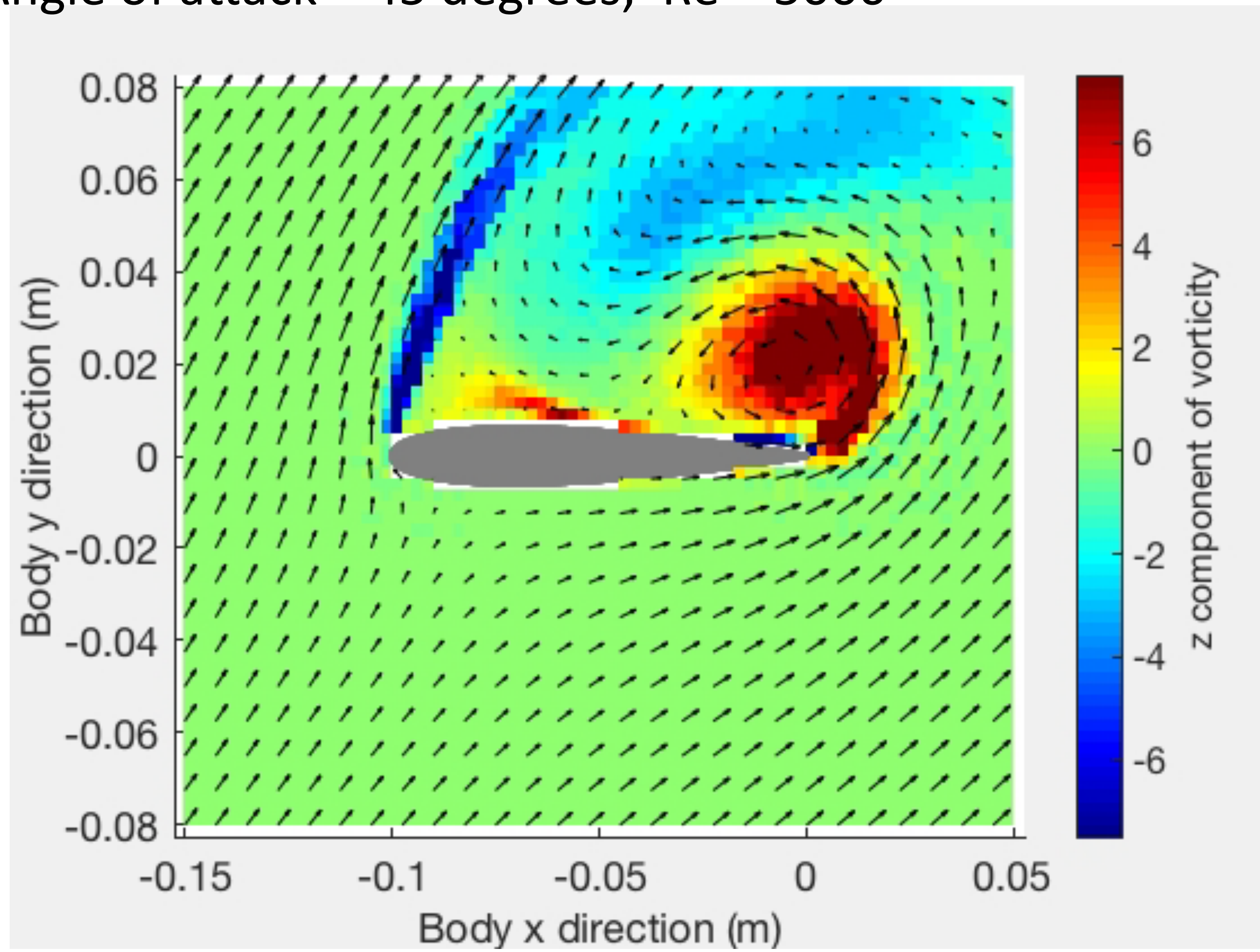
Separated flow at a high angle of attack (Source: Wikipedia)



Understanding the current state of the flow field and prominent flow structures is crucial for improved performance at high angles of attack

Computational Fluid Dynamics Simulation

- Angle of attack = 45 degrees; Re = 5000



Motivation for estimation research

- Control objective: lift maximization/regularization

- Flow estimation and control research has previously extracted only a few features of a flow for use in flow control applications (e.g., phase information for an oscillatory flow, etc.)

- In this application, lift *greatly* depends on the shedding of the periodic vortex structures, so it may be important to estimate the full flow field

- Recent advances in operator theoretic estimation and control approaches, such as using the *Koopman operator* and *Dynamic Mode Decomposition* (DMD), allow for simple, reduced-order models of highly nonlinear dynamics

Result: It may be possible to use high-fidelity fluid mechanical models in real-time estimation and control loops

Outline

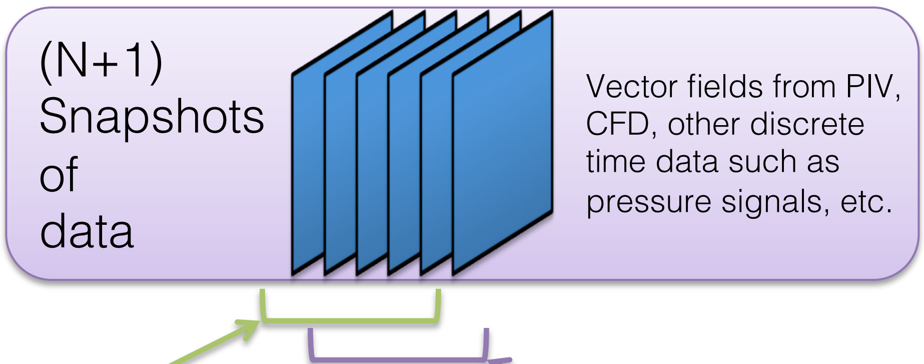
1. DMD and the Koopman operator
2. Sparsity-promoting DMD for a reduced-order model
3. DMD-based Kalman filtering: Flow over an airfoil

Dynamic Mode Decomposition (DMD)

- Originally formulated in the fluids literature (Schmid 2010), subsequently generalized (Tu et al. 2014)

$$\Psi = U\Sigma W^H$$

Principal Orthogonal Decomposition (POD) modes



$$\Psi = \begin{bmatrix} | & | & & | \\ \psi_0 & \psi_1 & \dots & \psi_{N-1} \\ | & | & & | \end{bmatrix} \qquad \Psi' = \begin{bmatrix} | & | & & | \\ \psi_1 & \psi_2 & \dots & \psi_N \\ | & | & & | \end{bmatrix}$$

- Look for the best-fit linear operator that marches the system forward

$$\Psi' \approx A\Psi$$

$$\underset{A}{\text{minimize}} \quad \|\Psi' - A\Psi\|_F^2 \quad \Longrightarrow \quad A = \Psi'\Psi^\dagger$$

(Tu et al. 2014)

DMD in POD-basis subspace

- *<u>If the state dimension is large,</u>* A *may be intractable to form*
- Search for $\tilde{A} \in \mathbb{C}^{r \times r}$, a version of A projected on the POD basis of snapshot matrix Ψ

$$A = U \tilde{A} U^H$$

- Optimization:

$$\underset{\tilde{A}}{\text{minimize}} \quad \|\Psi' - U \tilde{A} U^H \Psi\|_F^2 \qquad \Longrightarrow \qquad \tilde{A} = U^H \Psi' W \Sigma^{-1}$$

(Jovanović et al. 2014)

Snapshot reconstruction with DMD modes

- Eigendecomposition of $\tilde{A} = Y \Lambda Z^H$

Scaled version of Y so that
$Z^H Y = I$

$$A = U \tilde{A} U^H \quad \Longrightarrow \quad A = UY \Lambda Z^H U^H$$

Matrix of DMD eigenvalues

Matrix of DMD modes $\triangleq V$

- Snapshot dynamics

$$\left.\begin{aligned} q_{k+1} &= \tilde{A} q_k \\ \psi_k &\approx U q_k \end{aligned}\right\} \quad \begin{aligned} \psi_k &\approx U (Y \Lambda Z^H)^k U^H \psi_0 \\ &= V \Lambda^k \boxed{Z^H U^H \psi_0} \\ &= V \Lambda^k \alpha \end{aligned}$$

Vector of weighting coefficients dependent on initial condition

(Jovanović et al. 2014)

Koopman operator

- Nonlinear, discrete-time dynamics:

$$x_{k+1} = F_t(x_k)$$
$$y_k = g(x_k)$$

- Koopman operator (B. O. Koopman 1931)

$$\mathcal{K}_t \; g_j(x_k) = g_j(F_t(x_k)) = g_j(x_{k+1})$$

- Infinite-dimensional, linear operator that pushes forward an observable function under the dynamics

(Kutz et al. 2016)

Koopman operator (cont'd)

- Spectral representation:

$$\mathcal{K}_t\ \phi_j = \lambda_j \phi_j \quad \text{for} \quad j = 1, \ldots, \infty$$

Koopman eigenfunction

Koopman eigenvalue

$$g(x_k) = \sum_{j=1}^{\infty} v_j \phi_j(x_k)$$

Koopman mode

- Iteratively applying the Koopman operator and eigenfunction relations

$$\phi_j(x_k) = \mathcal{K}_t\ \phi_j(x_{k-1}) = \lambda \phi_j(x_{k-1}) = \cdots = \lambda^k \phi_j(x_0)$$

$$g(x_k) = \sum_{j=1}^{\infty} v_j \lambda_j^k \phi_j(x_0)$$

(Kutz et al. 2016)

Koopman-DMD Connection

Koopman

$$g(x_k) = \sum_{j=1}^{\infty} v_j \lambda_j^k \phi_j(x_0)$$

Consider only a finite number of modes

$$g(x_k) \approx \sum_{j=1}^{m} v_j \lambda_j^k \phi_j(x_0)$$

In finite dimensions, write in matrix form

$$g(x_k) \approx V \Lambda^k \phi(x_0)$$

DMD

$$\psi_k \approx V \Lambda^k \alpha$$

Correspondences

V, Λ Shared

$g(x_k) \longleftrightarrow \psi_k$

$\phi(x_0) \longleftrightarrow \alpha$

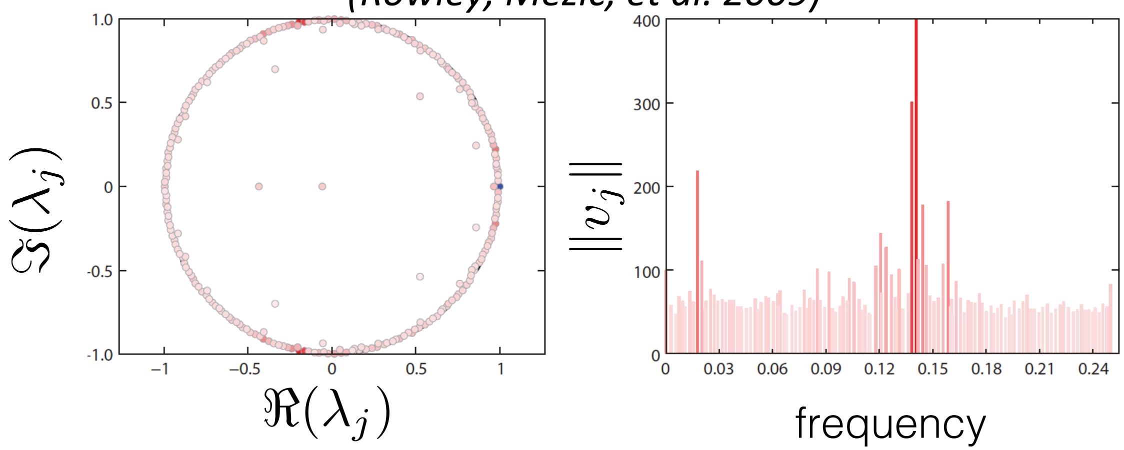
$\phi(x_k) \longleftrightarrow \Lambda^k \alpha$

DMD provides numerical estimates of the Koopman modes and Koopman eigenvalues. The weighting coefficients are the Koopman eigenfunctions at the initial time.

(Rowley, Mezic et al. 2009, Mezic 2013)

Reduced-order modeling with DMD

(Rowley, Mezic, et al. 2009)



- Select the “most important” modes for a reduced-order model
- Reduced-order model: (V, Λ)
- Reconstruction: Need to solve for the vector of weighting coefficients (DMD mode amplitudes) α

Matrix containing e-values raised to powers

$$\underset{\alpha}{\text{minimize}}\; J(\alpha) = \|\Psi - V D_\alpha V_{\text{and}}\|_F^2$$

Diagonal matrix containing α

- Observation from practice: Using only a few of the largest modes does not always lead to the best reconstruction!

Sparsity-Promoting DMD (SPDMD)

- Which modes should be included in a reduced-order model?
  - SPDMD automatically chooses the most relevant modes and their amplitudes
  - Example result:

$$\alpha = \left[\; \ldots \quad 0, \quad 0, \quad a, \quad 0, \quad 0, \quad \ldots \quad 0, \quad 0, \quad b, \quad c, \quad 0, \quad 0, \quad \ldots \; \right]^T$$

- Two-step optimization process:

Step 1: Optimize sparsity structure

$$\underset{\alpha}{\text{minimize}} \quad J(\alpha) + \gamma \underbrace{\sum_{j=1}^{r} |\alpha_j|}_{l_1 \text{ norm}}$$

Scalar weighting factor

Step 2: Amplitude “polishing”

$$\begin{aligned} &\underset{\alpha}{\text{minimize}} \quad J(\alpha) \\ &\text{subject to} \quad E^T \alpha = 0 \end{aligned}$$

Matrix enforcing sparsity structure

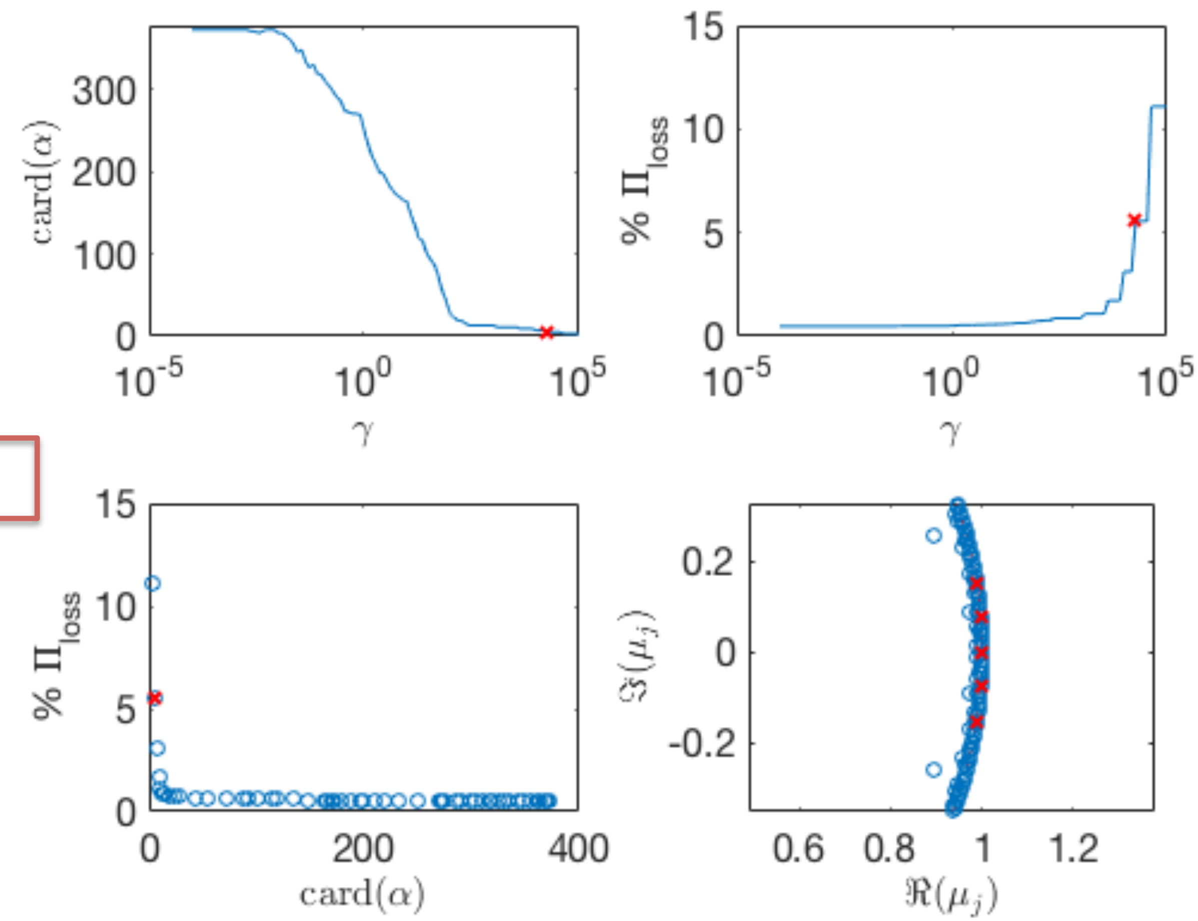
(Jovanović et al. 2014)

SPDMD results on flow over airfoil

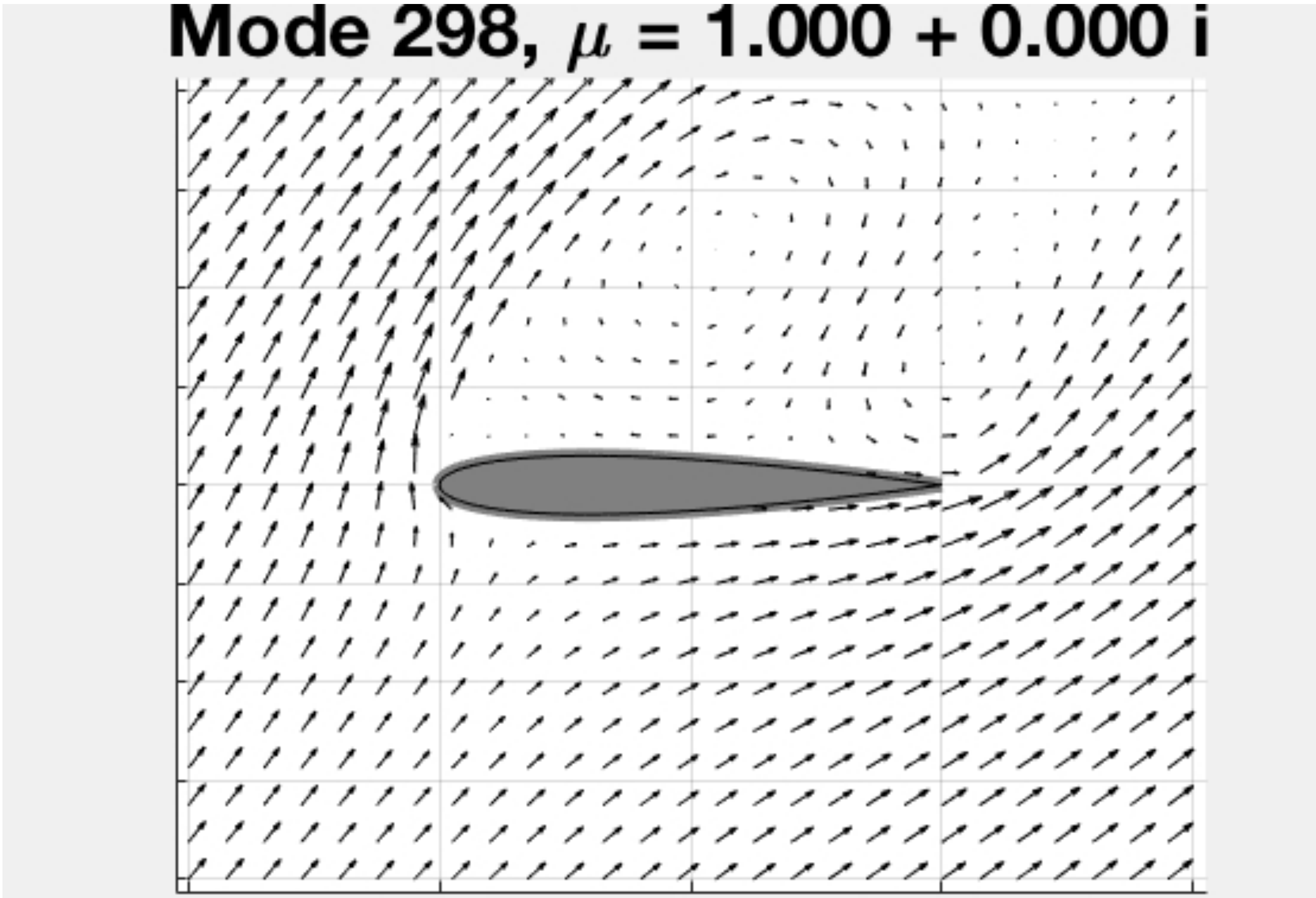
- The SPDMD result is a series of graphs for various regularization values from which one may choose the modes to retain
- Each point on the curve has a corresponding polished set of mode amplitudes

$$\%\Pi_{\text{loss}} = 100\sqrt{\frac{J(\alpha)}{J(0)}}$$

Size of original movie squared Ψ

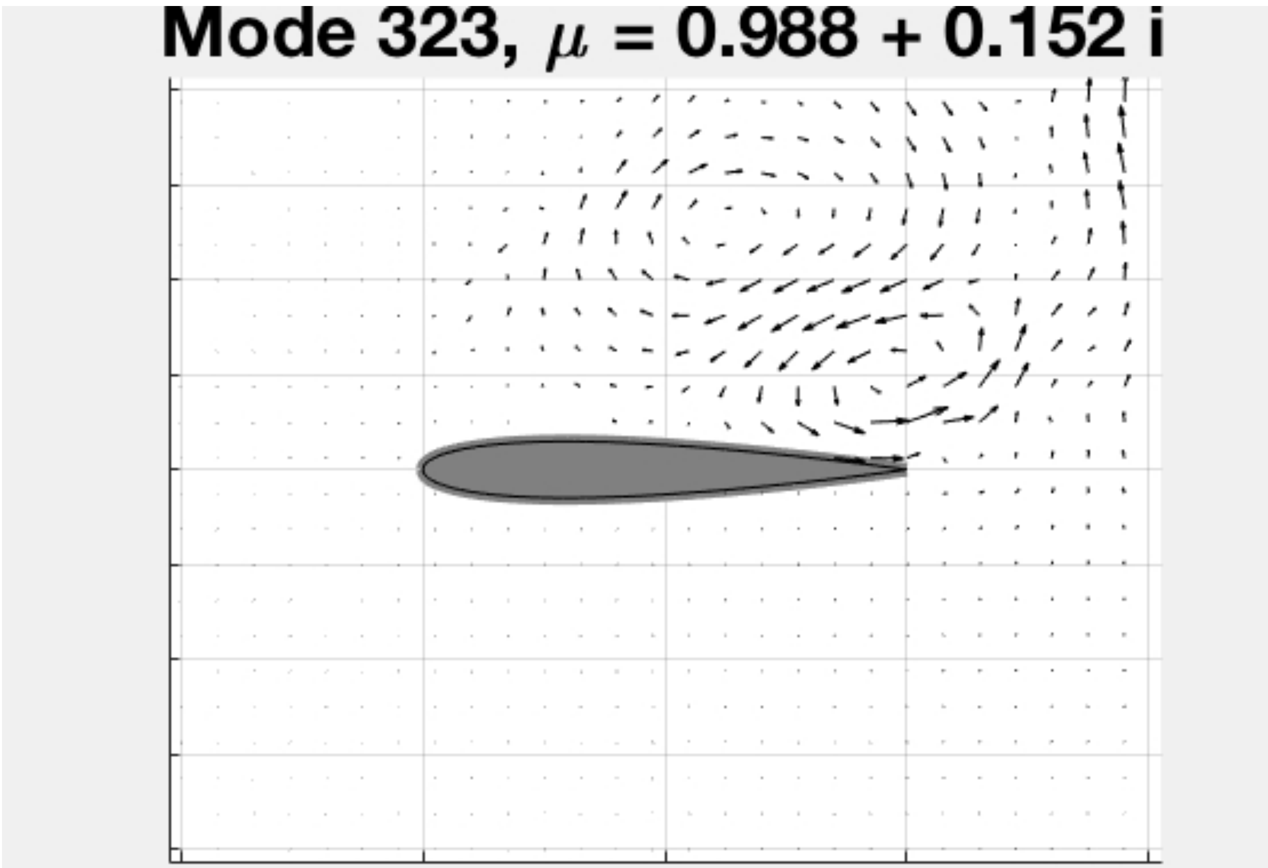


SPDMD-model mode 1 of 5



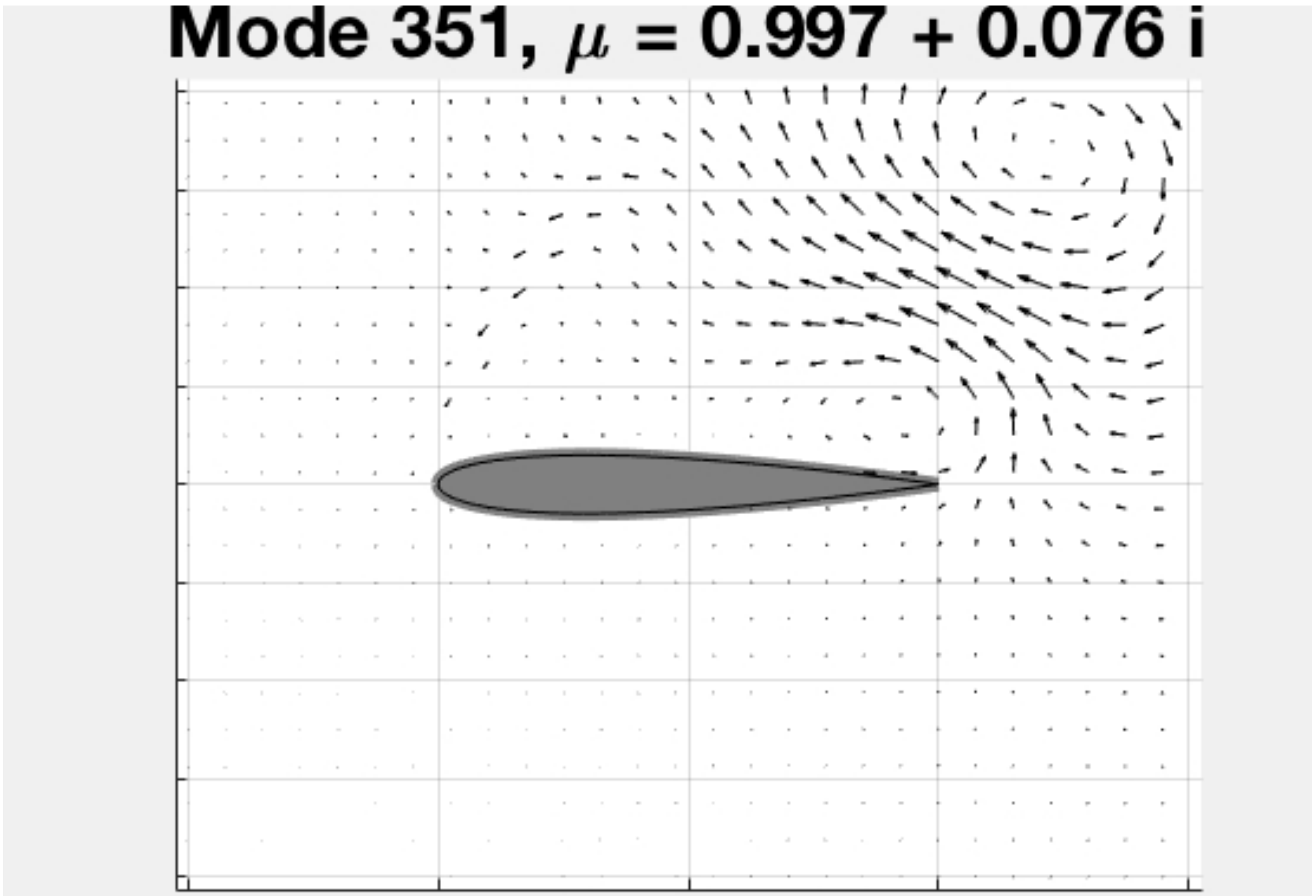
(Vorticity field suppressed for clarity)

SPDMD-model modes 2,3 of 5



(Vorticity field suppressed for clarity)

SPDMD-model modes 4,5 of 5

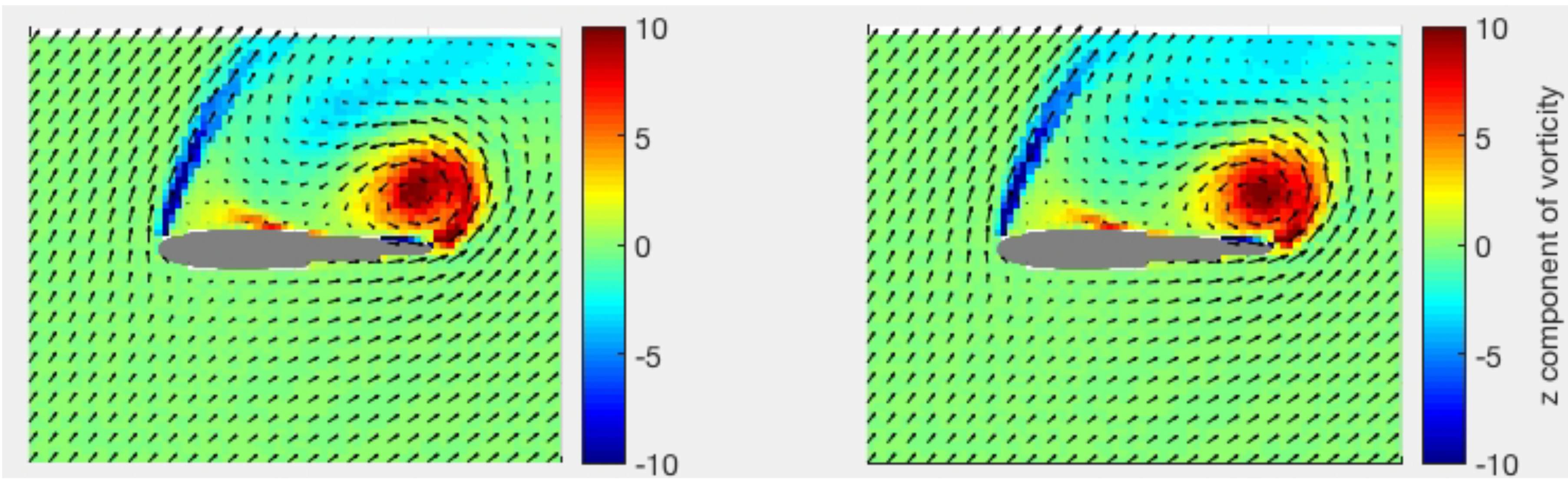


(Vorticity field suppressed for clarity)

Reconstruction of simulation

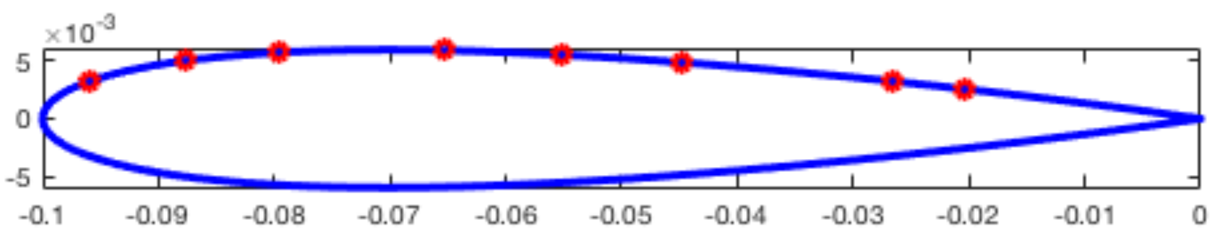
Original simulation

Reconstruction using 5 of 372 modes



DMD-based Kalman filtering experiment

- Distributed pressure sensors along the suction side of the airfoil
- Performed CFD simulations of ground truth and corrupted the sensor signals with additional white noise (5% signal mean value)



(Pressure sensor locations on NACA0012 airfoil)

Extraction of Koopman information

- Using CFD simulations, create a training data set
- Arrange state variables and outputs in composite snapshots

$$\Psi = \begin{bmatrix} | & | & & | \\ x_0 & x_1 & \cdots & x_{N-1} \\ | & | & & | \\ \\ | & | & & | \\ y_0 & y_1 & \cdots & y_{N-1} \\ | & | & & | \end{bmatrix} \qquad \Psi' = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_N \\ | & | & & | \\ \\ | & | & & | \\ y_1 & y_2 & \cdots & y_N \\ | & | & & | \end{bmatrix}$$

- Perform SPDMD calculation

Reduced-order, linear models from DMD

- Key observation: View Koopman eigenfunctions $\phi(x_k)$ as a transformation of the state vector x_k

$$z_k = \left[\begin{array}{ccc} \phi_1(x_k) & \ldots & \phi_M(x_k) \end{array} \right]^T$$

- Separate the Koopman modes into state and output portions

$$C^x = \left[\begin{array}{ccc} v_1^x & \ldots & v_M^x \end{array} \right] \qquad C^h = \left[\begin{array}{ccc} v_1^h & \ldots & v_M^h \end{array} \right]$$

Resulting linear system:

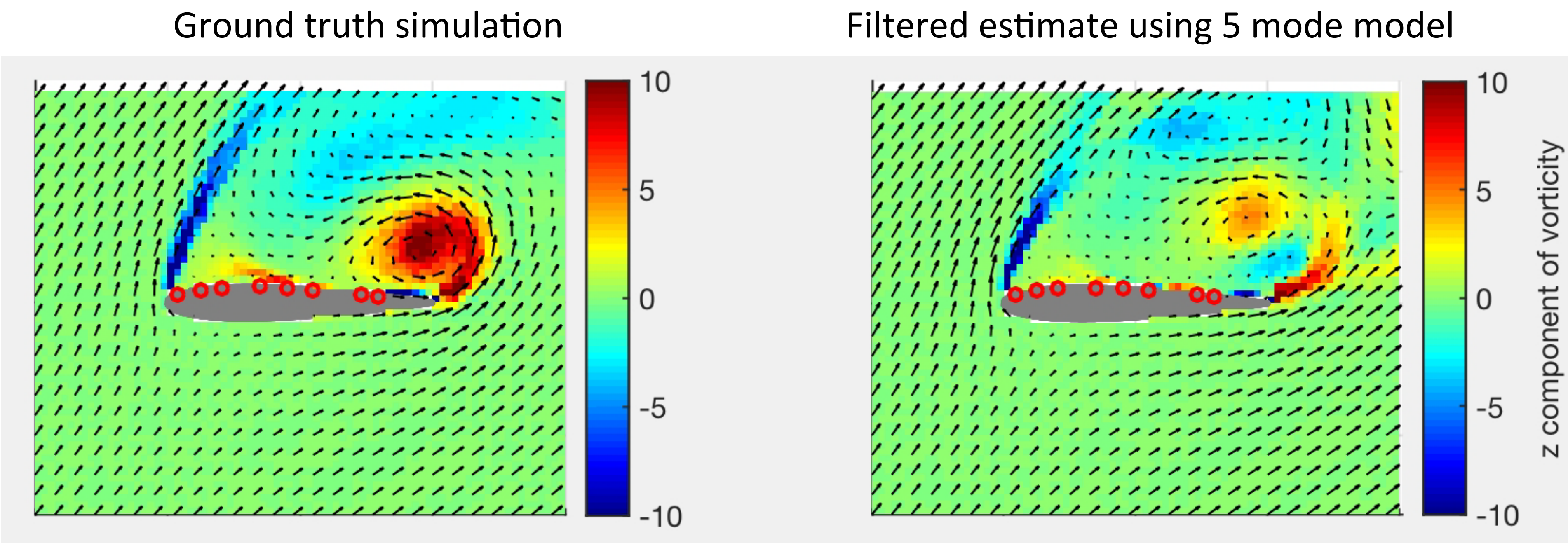
$$\boxed{\begin{aligned} z_k &= A z_{k-1} \\ y(x_k) &= C^h z_k \end{aligned}}$$

To transform back:

$$x_k = C^x z_k$$

- Note: The signals are complex-valued, so a complex filter is necessary. Otherwise, handle real and imaginary components separately with care (see Surana and Banaszuk 2016)

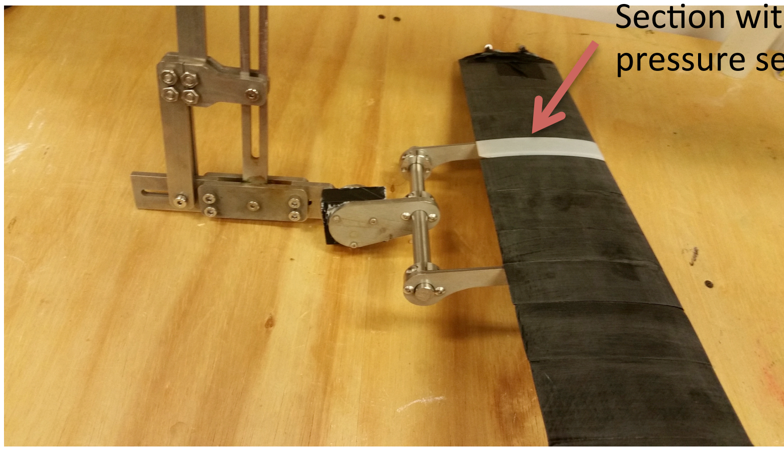
Pressure measurement filtering results



- *Initial transient required before estimate converges*
- *Estimation of phase and magnitude of vortex shedding well-captured*

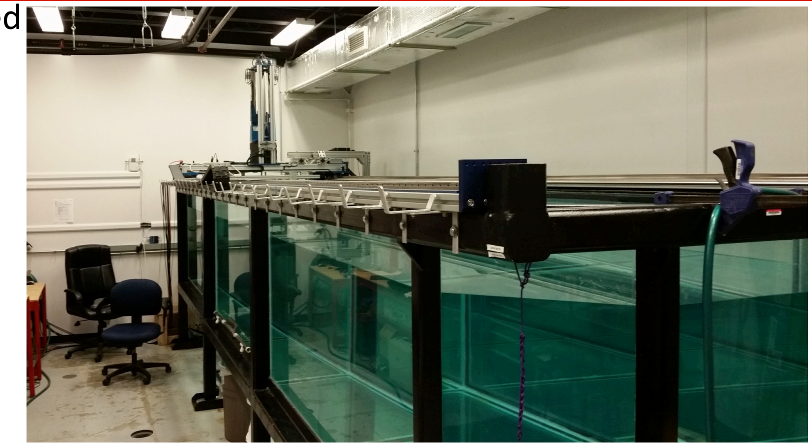
- *SPDMD permits rapid estimation with reduced model complexity; this approach could contribute to real-time control in future work*

Ongoing work: Experimental implementation

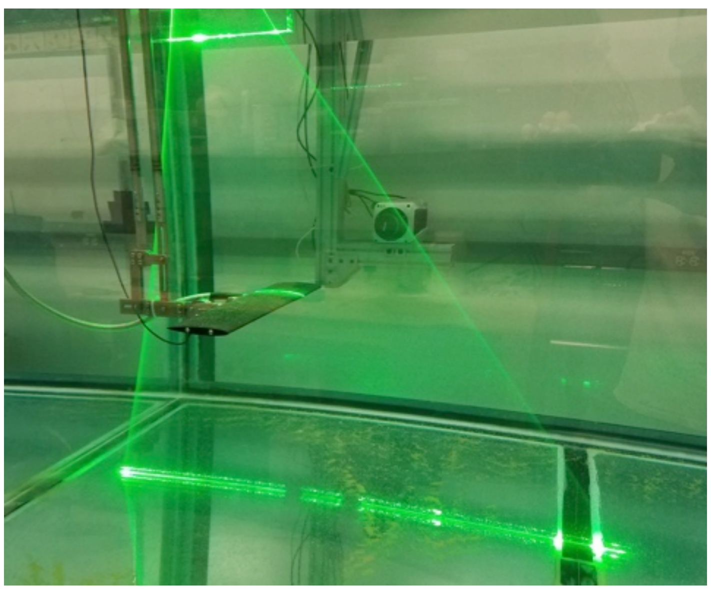


Section with embedded pressure sensor array

NACA 0012 wing model, courtesy of Phil Kirk



Tow tank in the lab of Prof. Anya Jones



Particle Image Velocimetry (PIV) data collection

<u>Objective:</u> Perform DMD-based Kalman filtering in real-time, using Koopman modes learned from simulation. Compare offline to estimate to the ground truth flow field collected using PIV.

Conclusion

- Koopman/DMD perspective has proven to be a promising approach for estimation of the flow behind a thin airfoil at high angles of attack in simulation
- Ongoing work:
  - Experimental implementation
  - Observability-based sensor placement
- Future work:
  - DMD-based real-time feedback control for lift regularization

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